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Extended abstract

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Optimal design of hardening and softening resonances of plane frame structures

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Summary. Nonlinear vibrations have shown many promising applications in the fields of micro- and nano-resonators. One well known phenomenon is the hardening/softening resonances. Here a structural optimization method is applied to tailor the hardening/softening nonlinearity of plane frame structures. A finite element model based on a geometrically exact beam element is used in the computational procedure and the design parameterization is facilitated by a variable thickness distribution. Sensitivities of the resonant amplitude and frequency of hardening/softening resonances with respect to design variables are calculated using the adjoint method. Examples are provided to demonstrate the possibility for controlling the hardening/softening behavior. Currently an extension to topology optimization of 2D continuum structure is in progress.

Introduction

Nonlinear vibrations have shown many promising applications in the fields of micro- and nano-resonators [1]. Also, the hardening and softening nonlinear resonances can be used to broaden the bandwidth of energy harvesters [2]. Among these researches, Cho et al. theoretically and experimentally showed that a micro-mechanical cantilever could display either hardening or softening resonance by controlling the geometric nonlinearity originating from a nanotube [3], and Dai et al. experimentally demonstrated that topological variation of an energy harvesting device could tune the bandwidth of hardening resonances [4]. Based on a topology optimization methodology [5], a systematic procedure for tailoring the hardening and softening nonlinear resonances is proposed.

Methods

A systematic optimization procedure is realized through design iterations using a nonlinear finite element model, nonlinear vibration analysis, sensitive analysis and gradient-based design updates.

Finite element model

The model of plane frame structure is implemented using the nonlinear beam element based on geometric exact beam theory [6], which takes into account large displacement, large rotation, shear deformation and rotational inertia. The current configuration of beam element is described as $\mathbf{x} = [x_0 + u \; v \; \varphi]^T$, where x_0 is the initial configuration in the axial direction, u is the displacement in axial direction, v is the deflection, and φ is the rotation angle. The strain-displacement relation is derived using the principle of virtual work leading to the following strain measures for the axial strain ϵ, the shear strain γ and the curvature κ:

$$\epsilon = (1 + \frac{du}{dx_0}) \cos\varphi + \frac{dv}{dx_0} \sin\varphi - 1, \quad \gamma = \frac{dv}{dx_0} \cos\varphi - (1 + \frac{du}{dx_0}) \sin\varphi, \quad \kappa = \frac{d\varphi}{dx_0} \tag{1}$$

In the finite element model the tangent stiffness matrix and internal force vector are derived based on the strain-displacement relations. The mass matrix is derived in a consistent manner taking into account rotational inertia. A damping matrix proportional to mass matrix and stiffness matrix is included as well. Both two-node and three-node beam elements are considered.

Nonlinear vibration analysis

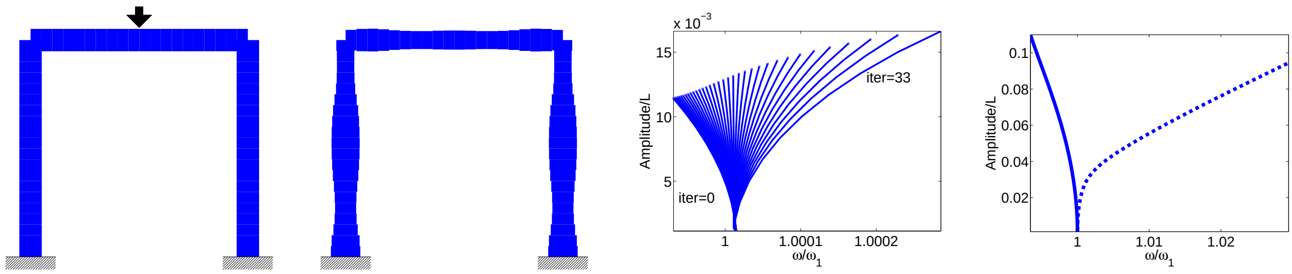
Nonlinear vibration analysis of the hardening and softening resonances is carried out by the Alternating Frequency/Time (AFT) domain method [7]. The AFT method is similar to the classical Incremental Harmonic Balance (IHB) method in the sense that all the governing equations are built and solved in the frequency domain. The primary difference is that AFT method transfers the responses solved in frequency domain to time domain, evaluates nonlinear terms (in this case in tangent stiffness and internal force) in time domain, and then transfers these nonlinear terms back to frequency domain which facilitates the treatment of the $\sin$ and $\cos$ nonlinear terms. Additionally, an advantage of the AFT method is that it can work interactively with general finite element codes and therefore can be added to the existing finite element codes as an extension.

Optimization formulation

Based on the results of the analysis, the sensitivities of the hardening and softening resonances with respect to the structural design variables are derived and calculated using the adjoint method, and gradient-based optimization is used to update the design variables [5]. The updated design is used in a new iteration and the procedure is repeated until a termination criterion is satisfied. For shape optimization of the frame structure, the optimization formulation to tailor the hardening and softening nonlinear resonances can be written as

$$
\begin{aligned}
&\min_{\rho_e} \ \pm\gamma \\
&\text{s.t. :}\ \gamma = \frac{\omega^*}{\omega_L}, \quad \mathbf{K}_L\mathbf{\Phi} = \omega_L^2\mathbf{M}\mathbf{\Phi}, \quad (\omega^*)^2\bar{\mathbf{M}}\bar{\mathbf{q}} + (\omega^*)\bar{\mathbf{C}}\bar{\mathbf{q}} + \bar{\mathbf{g}} = \bar{\mathbf{f}}, \quad b_{i1} = 0, \\
&\sum_{e=1}^{N_e} h_e \le \alpha N_e h_{max}, \quad h_e = h_{min} + \rho_e(h_{max} - h_{min}), \quad 0 \le \rho_e \le 1.
\end{aligned}
\tag{2}
$$

where ω^* is the nonlinear resonant frequency at specified load, ω_L is the linear resonant frequency, γ measures the extent of the hardening or softening nonlinearity, $h_e(e = 1,\dots,N_e)$ are the thicknesses of beam elements, $\rho_e(e = 1,\dots,N_e)$ are design variables used in the optimization and α represents a maximum allowable volume of the optimized beam. Note that b_{i1} is the coefficient of one sinusoidal term and $b_{i1} = 0$ is related to the phase lag quadrature criterion [8]. The barred matrices and vectors are in the frequency domain and can be found in [9]. ms with uniform cross sections with width 30 mm, thickness 30 mm and a slenderness ratio of 20. The material properties are $E = 2.05 \times 10^5\mathrm{MPa}, \nu = 0.3, \rho = 7800\mathrm{kg/m}^3$. The initial design has a softening resonance around the second linear vibration mode. The optimization problem is then to maximize the ratio γ by varying the thickness distribution. The design domain is bounded so that the thickness can vary from 20 mm to 40 mm. Besides, the total volume is constrained to not exceed half of the allowed volume and this causes the optimized non-uniform design have the same volume as the initial design. The optimized frame with non-uniform thickness, the evolution of the backbone during the iterations and comparison of initial backbone and optimized backbone at larger load are shown in Figure 1, which illustrates that the resonance has been turned from softening to hardening by optimizing the thickness distribution of the three beams.



(a) (b) (c) (d)

Figure 1: (a) Optimization problem, (b) optimized design, (c) evolution of backbone during iterations and (d) comparison of initial backbone and optimized backbone at larger load. solid line: initial design; dashed line: optimized design.

Conclusions

A systematic procedure is proposed to optimize the nonlinear resonances with hardening and softening behavior by using structural optimization to tune the geometric nonlinearity. Examples are given to demonstrate the effectiveness of the proposed procedure. This procedure can be applied to complex frame structures. Furthermore, the procedure can integrate other nonlinearities and other demands such as the desired frequency range in the optimization. The work is currently being extended to 2D continuum structures. The work was supported by the ERC starting grant INNODYN.

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